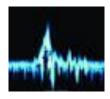
OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING



ECEN/MAE 5513 Stochastic Systems Fall 2011 Final Exam



PICK FIVE OUT OF ALL SIX PROBLEMS

Please specify	which F	IVE prol	blems to be gr	aded:
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Nome				
Name : _				
E-Mail Address:				

Problem 1:

Let X be a Rayleigh random variable with a=0. Find the probability that X will have values larger than its mode.

Please note from Homework Assignment #3:

<u>Problem 5</u>: Verify that the maximum value of $f_X(x)$ in which

$$f_X(x) = \frac{2}{b}(x-a)e^{-(x-a)^2/b}u(x-a)$$

for the Rayleigh density function occurs at $x = a + \sqrt{b/2}$ and is equal to $\sqrt{2/b} \exp(-1/2) \approx 0.607 \sqrt{2/b}$. This value of x is called the *mode* of the random variable.

Problem 2:

A random variable X undergoes the transformation Y = a/X, where a is a real number. Find the density function of Y, $f_Y(y)$.

Please note
$$F_Y(y) = P\{Y \le y\} = P\{x | Y \le y\} = \int_{\{x | Y \le y\}} f_X(x) dx$$

Problem 3:

The locations of hits of darts thrown at a round dartboard of radius r are determined by a vector random variable with components X and Y. The joint density of X and Y is uniform, that is

$$f_{X,Y}(x,y) = \begin{cases} 1/\pi r^2, & x^2 + y^2 < r^2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the densities of *X* and *Y*.

Problem 4:

For two statistically independent random variables *X* and *Y* show that

$$P{Y \le X} = \int_{-\infty}^{\infty} F_Y(x) f_X(x) dx$$

or

$$P\{Y \le X\} = 1 - \int_{-\infty}^{\infty} F_X(y) f_Y(y) dy.$$

Problem 5:

Consider random processes

$$X(t) = A\cos(\omega_0 t + \Theta)$$

$$Y(t) = B\cos(\omega_1 t + \Phi)$$

where A, B, ω_1 , and ω_0 are constants, while Θ and Φ are statistically independent random variables each uniform on $(0,2\pi)$.

- a) Show that X(t) and Y(t) are jointly wide-sense stationary.
- b) If $\Theta = \Phi$, show that X(t) and Y(t) are not jointly wide-sense stationary, unless $\omega_1 = \omega_0$.

Problem 6:

A Gaussian random process has an autocorrelation function

$$R_{XX}(\tau) = 6 \frac{\sin(\pi \tau)}{\pi \tau}.$$

Determine a covariance matrix for the random variables X(t), X(t+1), X(t+2) and X(t+3).